$\square$ Code No. : 21812

## VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD M.E. (Mech. Engg.: CBCS) I-Semester Main Examinations, January-2018

(Advanced Design \& Manufacturing)
Mathematical Methods for Engineers
Time: $\mathbf{3}$ hours
Max. Marks: 60
Note: Answer ALL questions in Part-A and any FIVE from Part-B

| Q. No | Stem of the Question | M | L | CO | PO |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Part-A (10 $\times 2=20 \mathrm{Marks}$ ) |  |  |  |  |
| 1. | Find the directional derivative of $\emptyset=x^{2} y z+4 x z^{2}$ at $(1,-2,1)$ in the direction of $2 \mathbf{i}-\mathbf{j}-2 \mathbf{k}$. | 2 | 1 | 1 | 10 |
| 2. | If $u \mathbf{F}=\nabla \mathrm{v}$ where u and v are scalar fields and $\mathbf{F}$ is a vector field, show that F.curl $\mathbf{F}=0$. | 2 | 2 | 1 | 10 |
| 3. | Define Solenoidal and Irrotational vectors. | 2 | 1 | 2 | 10 |
| 4. | Express the relationship between the associated tensors $\bar{A}^{i k l}$ and $A_{p q r}$ | 2 | 2 | 2 | 10 |
| 5. | Write the mathematical formula for Cramer's Rule. | 2 | 1 | 3 | 10 |
| 6. | Briefly explain LU decomposition. | 2 | 2 | 3 | 10 |
| 7. | Compute the Laplace transform of $e^{4 t} \sin 2 t$ cost. | 2 | 2 | 4 | 10 |
| 8. | Find the Laplace transform of $\sin$ at using basic definition. | 2 | 1 | 4 | 10 |
| 9. | Find the Fourier series of the function defined by $f(x)=\pi,-\pi \leq x \leq \pi$ | 2 | 2 | 5 | 10 |
| 10. | Find the Fourier series for the function defined by | 2 | 2 | 5 | 10 |
|  | $\begin{aligned} & f(x)=0,-\pi \leq x<0 \\ & f(x)=\sin x, 0 \leq x \leq \pi \end{aligned}$ |  |  |  |  |
|  | Part-B ( $5 \times 8=40$ Marks) |  |  |  |  |
| 11. a) | Show that $\nabla \times(\nabla \times \bar{A})=\nabla(\nabla \cdot \bar{A})-\nabla^{2} \bar{A}$ | 5 | 2 | 1 | 10 |
|  | Find the unit normal vector to $x^{2}+y^{2}+z^{2}=5$ at $(0,1,2)$ | 3 | 1 | 1 | 10 |
| 12. | Let $A_{r s t}^{p q}$ be a tensor. Find the rank of $A_{r s t}^{p q}$ when $\mathrm{p}=\mathrm{t}$ and $\mathrm{q}=\mathrm{s}$ | 5 | 2 | 2 | 10 |
| b) | Using tensor notation show that curl $\operatorname{grad} \varphi=0$ | 3 | 3 | 2 | 5 |
| 13. a) |  | 5 | 4 | 3 | 5 |
|  | $x^{(1)}, y^{(1)}, z^{(1)}, w^{(1)}, x^{(2)}, y^{(2)}, z^{(2)}, w^{(2)}$ <br> for the following system of equations |  |  |  |  |
|  | $3.49 x-0.25 y+9.21 z+0.05 w=1.32$ |  |  |  |  |
|  | $5.25 x-1.77 y+8.97 z+0.1 w=4.35$ |  |  |  |  |
|  | $\begin{aligned} & 1.73 x-2.1 y+3.37 z+7.23 w=12.49 \\ & 1.23 x-6.54 y+2.87 z+2.41 w=11.32 \end{aligned}$ |  |  |  |  |
|  | Superscripts indicate iterations. |  |  |  |  |
| b) | Find $a, b$ and $c$ so that the linear system $\begin{aligned} & x+2 y-3 z=a \\ & 2 x+3 y+3 z=b \\ & 5 x+9 y-6 z=c \text { is consistent. } \end{aligned}$ | 3 | 3 | 3 | 5 |

14. a) Solve the initial value problem $y^{\prime \prime}+4 y^{\prime}+3 y=e^{-t}, y(0)=y^{\prime}(0)=1$ by using Laplace transforms.
b) Find the inverse Laplace transforms of $\frac{4 s+5}{(s-1)^{2}(s+2)}$
15. a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is vibrating by giving to each of its end points a velocity $\lambda x(l-x)$, find the displacement of the string at any distance $x$ from one end at any time $t$.
b) Expand $f(x)$ in a Fourier series on the interval $-2 \leq x<2$ if $f(x)=0$ for $-2 \leq x<0$ and $f(x)=1$ for $0 \leq x<2$
16. a) Prove that $\operatorname{curl} \operatorname{curl} F=\operatorname{grad} \operatorname{div} F-\Delta^{2} F$
b) Verify the identity $A \cdot(B \times C)=B \cdot(C \times A)$ using tensor notation.
17. Answer any two of the following:
a) Find the Eigen values and corresponding Eigen vectors of matrix
$A=\left[\begin{array}{ccc}1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 4\end{array}\right]$
b) Find $L^{-1}\left\{\frac{\frac{s}{2}+\frac{5}{3}}{s^{2}+5 s+6}\right\}$
c) Given the non-orthogonal basis

$$
a_{1}=\mathbf{i}-\mathbf{j}-\mathbf{k}, a_{2}=\mathbf{i}+\mathbf{j}+\mathbf{k}, a_{3}=-\mathbf{i}+2 \mathbf{k}
$$

use the Gram-Schmidt orthogonalization process to find the orthonormal basis.

| 5 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: |
| 3 | 2 | 4 | 10 |
| 5 | 5 | 5 | 12 |
| 3 | 4 | 5 | 5 |
| 4 | 1 | 1 | 10 |
| 4 | 2 | 2 | 10 |
| 4 | 6 | 3 | 12 |
| 4 | 3 | 4 | 5 |
| 4 | 2 | 5 | 10 |

M: Marks; L: Bloom's Taxonomy Level; CO: Course Outcome; PO: Programme Outcome

| S. No. | Criteria for questions | Percentage |
| :---: | :---: | :---: |
| 1 | Fundamental knowledge (Level-1 \& 2) | 58.75 |
| 2 | Knowledge on application and analysis (Level-3 \& 4) | 30.00 |
| 3 | *Critical thinking and ability to design (Level-5 \& 6) |  |
| (*wherever applicable) | 11.25 |  |

